RELATIONSHIP BETWEEN GAIN AND NOISE FIGURE OF AN OPTICAL ANALOG LINK

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ABSTRACT

Experimental confirmation is presented of the relationship between link gain and noise figure by varying the optical attenuation in an external modulation link. When we minimized the optical loss we achieved an amplifierless link gain of 31 dB and noise figure of 4.2 dB at 150 MHz.

INTRODUCTION

In many applications of analog fiber optic links, the link noise figure and IM-free dynamic range are two of the most important parameters. While in principle any link noise figure can be overcome with a sufficiently high-gain pre-amplifier, the higher the link noise figure the less feasible this approach is in practice, especially when one is simultaneously trying to meet an IM-free dynamic range requirement. Consequently it is important to reduce the link noise figure to the lowest practical value. This raises the question as to what are the limits on the achievable link noise figure.

Two earlier papers [1,2] have referenced the noise figure limits. In [1] it was claimed that, under the constraint that the link input was passively and losslessly matched to the source, the minimum link noise figure, *NF*, was 3 dB. This lower limit of 3 dB was labeled as the *passive match limit* and was independent of the link gain, *G*. (We are using "gain" here in the most general sense; *i.e.*, it includes negative gain or loss.) Unfortunately, this limit has proven hard to achieve in practice because all realizable passive matching circuits have loss.

More recently [2] it was pointed out that in analog optical links with *NF* far above the passive match limit there appears to be a relationship between *G* and *NF*. In Figure 1 (from [2]) a collection of experimental data points gathered from a review of published link measurements provide experimental support for what we call the *passive attenuation limit*. None of these data violate the passive attenuation limit; however neither do they support the details of this limit, because none of the individual links was measured at more than one value of *G-NF*.

Thus the goals of the present work are to derive these two limits analytically and to validate the limits experimentally. In the process a third limit which takes into account the matching circuit loss is introduced.

THEORETICAL DEVELOPMENT

To facilitate an examination of the *NF* limits we use an analytical model for the performance of an intensity-modulated link in which the modulator or directly-modulated laser and the photodetector have been perfectly impedance-matched to the link input and output, respectively, using passive circuits.

A. Lossless Passive Match Limit

The noise figure of any two-port device is defined, according to IEEE standards [3], as "the ratio of the available output noise power per unit bandwidth to the portion

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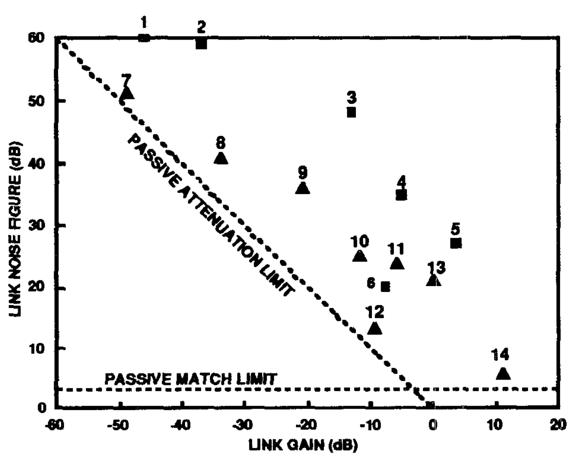

■ DIRECT MODULATION		
POINT NO.	FREQ. (GHz)	REFERENCE
1	11.0	WESTINGHOUSE
2	9.0	LASERTRON
3	12.0	GE
4	1.5	MIT LINCOLN LAB
5	0.9	GE
6	1.8	MIT LINCOLN LAB

▲ EXTERNAL MODULATION		
POINT NO.	FREQ. (GHz)	REFERENCE
7	10.0	NAVY
8	20.0	MIT LINCOLN LAB
9	3.9	UT PHOTONICS
10	10.0	MARTIN MARIETTA
11	0.8	MIT LINCOLN LAB
12	0.9	GE
13	0.9	GE, DREXEL U.
14	0.06	MIT LINCOLN LAB

Fig. 1. Published insertion gains and noise figures for experimental analog links (after [2]).

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of that noise caused by the actual source connected to the input terminals of the device, measured at a standard temperature of 290 °K".

The available link output noise power per unit bandwidth can be expressed as the sum of three components. One component of the link output noise is caused by the thermal noise of the source connected to the link input: $kT \times G$, where k is Boltzmann's constant and T=290 °K. Whatever the modulation device at the link input, whether external modulator or diode laser, this device has a physical (ohmic) resistance. When this resistance is perfectly impedance-matched to the source using a lossless passive circuit, it also contributes $kT \times G$ to the link available output noise. The third component of the link output noise consists of optical noise detected and thermal noise generated by the photodetector circuit. Minimum link NF is obtained when the link G is sufficiently high to cause this third noise power to be negligible. Thus, according to the NF definition, the minimum attainable NF for the link is:

$$\lim_{G\to\infty} NF = \lim_{G\to\infty}\left(10\log\left[\frac{kT\,G + kT\,G}{kT\,G} + \frac{\text{detector terms}}{kT\,G}\right]\right) = 3\,\text{dB}, \qquad (1)$$

which is the *lossless passive match limit* to link NF.

B. Passive Attenuation Limit

In the general case, the optically generated noises in the photodetector term of equation (1) are not negligible, and thus preclude a link NF approaching the 3 dB lossless passive match limit, as Figure 1 attests. For an intensity-modulated optical link having passive lossless input and output impedance-matching circuits, an expression for NF was derived in [1] that included the effects of detector shot noise and laser relative intensity noise (RIN):

$$NF = 10\ \log\left[2 + \frac{N_D^2\,R_{in}}{kT\,G}\left(i_{sn}^2 + i_{RIN}^2\right)\right], \qquad (2)$$

where N_D is the turns ratio of the lossless impedance transformer in the detector matching circuit, R_{in} is the system source and load impedance, and i^2_{sn} and i^2_{RIN} are the spectral densities of the shot and RIN photocurrents.

In the original derivation of equation (2), the photodetector thermal noise was neglected because in the cases considered in that paper this noise was swamped by the shot noise and RIN terms. However, to derive the passive attenuation limit we must include this noise source. In the detector's lossless, passive impedance-matching circuit, the physical resistances in the photodetector cause an additional noise power per unit bandwidth of kT to appear at the link output. Again using the NF definition above, this contributes to the noise figure an additional term of $kT/(kT \times G)$, or $1/G$. Therefore,

$$NF = 10\ \log\left[2 + \frac{N_D^2\,R_{in}}{kT\,G}\left(i_{sn}^2 + i_{RIN}^2\right) + \frac{1}{G}\right]. \qquad (3)$$

When the detector noise terms are negligible—as they can be for large optical attenuation, for example—the detector thermal noise term emerges as the dominant noise source. Consequently the limiting case of equation (3) is:

$$NF = 10\ \log\left[2 + \frac{1}{G}\right]. \qquad (4)$$

The constant term 2 in equation (4) is the lossless passive match limit that was derived above. The $1/G$ term is referred to here as the *passive attenuation limit*, so named because it is the same result one obtains for a passive attenuator. The curves shown in Figure 2 are plots of equation (3) for the case where i^2_{RIN} is negligible compared to i^2_{sn}, and where $(N^2_D \times R_{in})$ =500 Ω. The solid line curve is for a link in which (i^2_{sn}/G) =0.0025 A^2/Hz at the optical loss that yields a link G of 0 dB. The dashed curve in Figure 2 represents the same curve with a factor of 10 higher modulation efficiency; it is essentially a plot of equation (4) with the shot and RIN noise terms set to zero. For all values of G<<0 on this curve NF approaches the passive attenuation limit of $1/G$. For all values of G>>0, the NF approahes the lossless match limit of 3 dB.

C. General Passive Match Limit

To compare these theoretical limits with experimental results will require the inclusion of matching circuit loss in the preceding limit equations. Also, in anticipation of the experimental confirmation of these limits, we will need to choose a link implementation that is capable of high link gain and has negligible RIN. Only external modulation links with solid-state lasers presently meet these requirements.

Figure 3 shows an equivalent circuit of the external modulation link. In the source-to-modulator and detector-

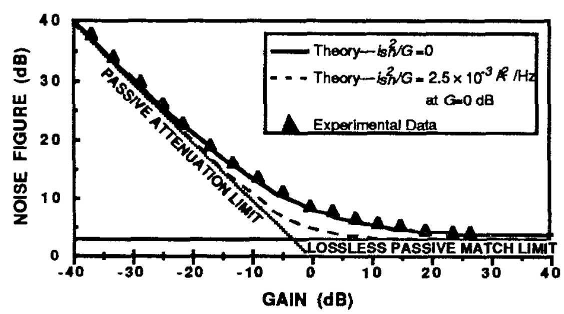
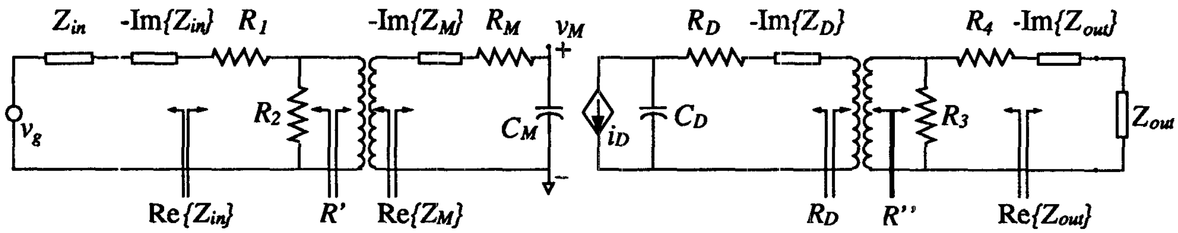


Fig. 2. Relationship between G and NF for an analog link with $(N_D^2 \cdot R_{in})$=500 Ω and negligible i_{RIN}. Dashed curve relates case with very high modulation efficiency, such that NF=2+1/G. Solid line is theoretically predicted NF for modulation efficiency of experimental external modulation link, in which i^2_{sn}/G=2.5×10^{-3} at G=0 dB. Measured data are also plotted.



For perfect passive match to modulator: $R_2 = \frac{R_{in}^2 - R_1^2}{R_1}$ *and* $R' = \frac{R_{in}^2 - R_1^2}{R_{in}}$

For conventional Mach-Zehnder modulator: $i_D = \frac{\eta_D G_{opt} P_L}{2V_\pi}\sin(\phi_{bias})$

For perfect passive match to detector: $R_3 = \frac{R_{out}^2 - R_4^2}{R_4}$ *and* $R'' = \frac{R_{out}^2 - R_4^2}{R_{out}}$

Fig. 3. Equivalent circuit of external modulation link. Resistors R_1, *etc.*, model the effect of matching circuit losses upon link G, NF.

to-load impedance-match circuits, the respective device's reactance is nulled at a single frequency using a series reactive element. The resulting purely real impedance is transformed to the port impedance using a combination of lossless impedance transformer and resistors.

Resistors R_1, R_2 in the modulator circuit (and R_3, R_4 in the detector circuit) model the effect of matching-circuit insertion losses upon link G and NF. The perfect match condition dictates fixed relationships among R_{in}, R_1, R_2, and R', such that all four resistances are expressed using only R_{in} and R_1. Similar relationships exist in the detector matching circuit; these are stated in Figure 3.

A straightforward evaluation of the circuit in Figure 3 yields the following equation for P_{out} relative to $P_{in,av}$:

$$P_{out} = G\, P_{in,av} = \frac{1}{\omega^2 C_M^2 R_M}\left(\frac{\eta_D G_{opt} P_L}{2V_\pi}\right)^2 N_D^2 R_{out} \times\left[\frac{R_{in}-R_1}{R_{in}+R_1}\right]\left[\frac{R_{out}-R_4}{R_{out}+R_4}\right]P_{in,av}, \quad (5)$$

in which the bracketed ratios equal the modulator and detector matching circuit losses G_M, G_D. In the zero-loss case, *i.e.* when R_1=R_4=0 and R_2=R_3=∞, equation (5) reduces to the previously derived [1] equation for output power with lossless matching. Now when we derive the thermal noise power densities at the link output due to resistors R_{in}, R_1, R_2 and R_M, and divide by noise due only to R_{in} (as per the NF definition [3]), the result can be expressed in terms of modulator matching loss G_M:

$$NF = 10\log\left[\frac{kTG + kTG\left(\frac{1-G_M}{1+G_M}\right)+\frac{kTG}{G_M}\left(\frac{1-G_M}{1+G_M}\right)+\frac{kTG}{G_M}}{kTG} + \frac{\text{detector terms}}{kTG}\right] = 10\log\left[\frac{2}{G_M}+\frac{\text{detector terms}}{kTG}\right]. \quad (6)$$

Adding the noise power densities due to shot noise, RIN, and ohmic sources R_D, R_3 and R_4 yields:

$$NF = 10\log\left[\frac{2}{G_M} + \frac{G_D N_D^2 R_{in}\left(i_{sn}^2 + i_{RIN}^2\right) + kTG_D + kTG_D\left(\frac{1-G_D}{1+G_D}\right) + kT\left(\frac{1-G_D}{1+G_D}\right)}{kTG}\right] = 10\log\left[\frac{2}{G_M} + \frac{G_D N_D^2 R_{in}}{kTG}\left(i_{sn}^2 + i_{RIN}^2\right) + \frac{1}{G}\right]. \quad (7)$$

Again, if the optical detector noises are negligible because of very high optical attenuation, very low V_π, or both, the limiting case of equation (7) is:

$$NF = 10\log\left[\frac{2}{G_M}+\frac{1}{G}\right]. \quad (8)$$

Note that the passive attenuation limit is still $1/G$; however, the passive match limit is now

$$\lim_{G\to\infty} NF = 10\ \log\left[\frac{2}{G_M}\right] = 3\,\text{dB} + 10\log\left[\frac{1}{G_M}\right], \quad (9)$$

rather than simply 3 dB. We call this the *general* passive match limit; it includes the zero-loss (*i.e.*, G_M=1) limit of 3 dB. The limit dictates that, in a direct or external modulation optical link with the input device passively matched to the source impedance, the lowest achievable NF is 3 dB plus the loss (in dB) in the matching circuit.

EXPERIMENTAL RESULTS

Figure 4 shows a block diagram of the experimental external modulation link we used to validate the theory. The principal components of the link were a high power, diode-pumped, 1.3-μm Nd:YAG laser, a Mach-Zehnder modulator formed in $LiNbO_3$ with Ti-indiffused waveguides, and an InGaAs photodiode. Short lengths of fiber

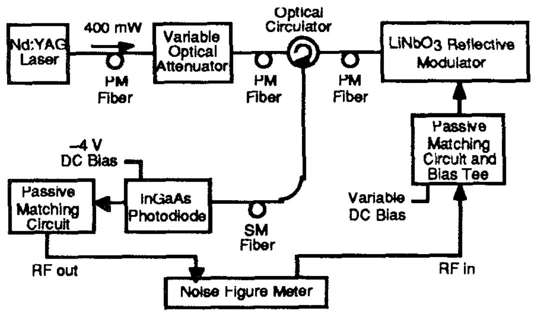


Fig. 4. Experimental external modulation optical link used for measuring the relationship between G and NF.

were used to connect the components, with polarization-maintaining single-mode fiber between the laser and the modulator and standard single-mode fiber between the modulator and the photodiode. To vary the link gain we used a variable optical attenuator with a range of >25 dB, which furnished a range in $G > 50$ dB.

To enable as high a maximum G as possible, we used a high average optical power, a low-V_π modulator, and photodetector impedance matching. We achieved low V_π by combining a reflective modulator—which doubled the effective electrode length, thereby halving V_π—with a resonant RF match to the electrodes at the frequency of interest (150 MHz in this case). Photodetector impedance matching, using a resonant match similar to the modulator's, contributed about 10.5 dB to the link gain.

With the optical attenuator set to zero, the optical loss between the input of the modulator and the detector was 7.9 dB. Consequently with 225 mW of fiber-launched laser optical power and the modulator set to the quarterwave bias (90°, or $V_\pi/2$), the average photocurrent at our detector (which had a responsivity of 0.85 A/W) was 15.5 mA. Under these conditions, the link's gain was 31.0 dB at 150 MHz, of which 10.5 dB was due to photodetector matching. As far as the authors are aware, this is the highest intrinsic gain reported to date—*i.e.*, for an amplifierless optical link.

Under these same conditions the noise figure was measured to be 4.6 dB; however, as the modulator bias was moved closer to the halfwave bias (180°, or V_π), the noise figure decreased to its minimum measured value of 4.2 dB, which is 1.2 dB greater than the 3 dB limit for an external modulation link with a modulator perfectly matched to the input using lossless reactive elements. The input matching circuit loss was measured separately and found to be 0.7 dB. Thus, in this maximum-gain state we expected to measure a minimum noise figure of 3.7 dB (3 dB + 0.7 dB) for the link.

Increasing the optical attenuation incrementally, we measured G and NF at each increment. The measured G-vs.-NF data were plotted in Figure 2. The bias point was set at 140° for all the data points shown. The solid curve in Figure 2 is the *general passive match limit* as calculated for the experimental parameters of this link. As can be seen for $G \gg 1$, the NF asymptotically approaches the 3.7 dB limit. For $G \ll 1$, the input thermal noise overpowers the shot noise, and the NF approaches the $1/G$ limit.

SUMMARY

This paper has proposed a general passive match limit for an intensity-modulated link: we derived it theoretically and verified it experimentally. The effects of passive matching loss were included and found to raise the 3 dB lossless asymptotic limit by the amount of the matching circuit loss. Further, it was noted that the greater the modulation device sensitivity, the more closely the link performance approaches the general passive match limit. Although the general passive match limit is applicable to both direct and external modulation links, the state of the art restricted experimental confirmation to a link in which a solid-state laser was externally modulated. In such a link, the *RIN* is negligible, and the gain can be made quite high by a suitable choice of link parameters.

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