Linearized Optical Modulator with Fifth Order Correction

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***Abstract*— A linearized solution for a series cascade of three Mach Zehnder interferometers separated by directional couplers is obtained that eliminates both the third and fifth order coefficients of the Taylor series representation of the transfer function. The device is characterized in terms of harmonic distortion with a three tone signal. ~20-dB improvement in third harmonic distortion is obtained compared to a two section device.**

I. Introduction

INTEGRATED optical modulators tend to have sinusoidal transfer functions (the dependence of the amplitude modulated beam on the applied voltage). These modulators are used in microwave transmission, RF field detection, and cable TV, all applications in which a linear transfer function would be preferred. Generally the modulators are biased to the linear portion of the sine curve and used with small signals to maintain linearity. The sine transfer function results in a limited linear dynamic range for the device. Because of this many authors have tried to modify the modulator to produce a more linear transfer functions [1]–[6]. If the sine transfer function is considered as an odd Taylor series expansion, proposals have been made to eliminate the third order Taylor series term [1], [2], [4], [6], or to balance the third and fifth order terms such that the contribution of a specific harmonic mixing term is reduced [5]. The even (second, fourth, etc.) Taylor series terms can be eliminated by appropriate biasing of the device. In this paper we consider a serial cascade of three Mach Zehnder interferometers, separated by directional couplers, and obtain a solution for which both the third and fifth order Taylor series terms are eliminated. This result should provide the most linear transfer function for an optical modulator obtained to date. The device is characterized in terms of harmonic distortion with a three tone signal.

In [1] a 1×2 modulator is proposed which consists of a serial cascade of a Y branch, two Mach Zehnder interferometers and two directional couplers [shown in Fig. 1(a)]. In this device a RF drive and appropriate DC biases are applied to each interferometer, but the coupling coefficients in each coupler are considered to be fixed values. A solution removing the third order Taylor series term was obtained under the condition of equal coupling coefficients for the directional couplers. The authors concluded that the RF drive to the second interferometer should be -0.5 times the drive to the first interferometer. In [6], where the same device is considered, the constraint on equal coupling is removed and the authors obtain an optimized solution with the RF drive to the second interferometer -1.0 times the drive to the first interferometer. They also derive the values for the two coupling coefficients which eliminate the third order Taylor series term. The latter solution is preferred because it results in a more efficient device, with respect to drive voltage, although the harmonic distortion characteristics of the two solutions are similar. This serial cascade approach has several advantages, as pointed out in [2] and [6]:

1) Drive voltages are applied to the interferometers only, which may be made traveling wave, so that the results are independent of frequency.
2) Asynchronism ($\Delta\beta$ mismatch) in the directional couplers can be compensated for by bias adjustment of the interferometers [6].
3) The improvement in linearity is independent of drive voltage.

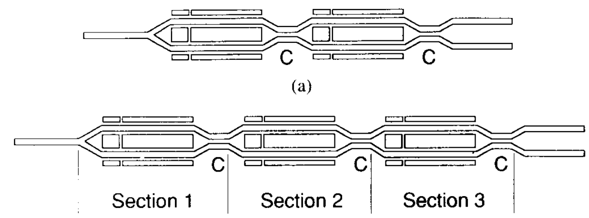

Fig. 1. (a) The two section serial cascade modulator of [2] and [6]. (b) The three section extension of Fig. 1(a). Shown are signal and bias electrodes on the interferometers. The directional couplers (C) may also be electrically tuned with electrodes not shown.

We propose to extend the device of Fig. 1(a) by adding an additional section (interferometer and directional coupler) as in Fig. 1(b). Following the result of [6] we expect to apply the RF drive voltage to the interferometers in the ratio 1:−1:1, i.e., apply equal magnitude voltages, but with alternating sign. The additional coupler C_3 should then provide an additional adjustable parameter which will allow the fifth, as well as the third, order Taylor series term to be removed. In principle this can be extended further, with each additional section allowing an additional order of distortion to be removed. Here we provide the solution for a three section device only.

II. Model

The output of the device of Fig. 1(b) can be obtained using coupled or normal mode theory in the usual way. We use

Manuscript received September 23, 1994; revised December 16, 1994.
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IEEE Log Number 9413042.

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normal mode theory [7] to obtain for the output optical power of one of the output channels

$$P = \tfrac{1}{2}(1 + A\cos\Delta\phi_{ij3} - B\sin\Delta\phi_{ij3}) \tag{1a}$$

where

$$\begin{aligned} A =& \cos\Delta\phi_{ij2}\sin\Delta\phi_{ij1}\sin\Delta\phi_{ab1} \\ &+ \sin\Delta\phi_{ij2}\cos\Delta\phi_{ab1}\sin\Delta\phi_{ab2} \\ &+ \cos\Delta\phi_{ij1}\sin\Delta\phi_{ij2}\sin\Delta\phi_{ab1}\cos\Delta\phi_{ab2} \quad (1b) \\ B =& -\sin\Delta\phi_{ab3}(\cos\Delta\phi_{ab2}\cos\Delta\phi_{ab1} \\ &- \sin\Delta\phi_{ab2}\sin\Delta\phi_{ab1}\cos\Delta\phi_{ij1}) \\ &+ \cos\Delta\phi_{ab3}(\sin\Delta\phi_{ij1}\sin{}_{ij2}\sin\Delta\phi_{ab1} \\ &- \cos\Delta\phi_{ij1}\cos\Delta\phi_{ij2}\sin\Delta\phi_{ab1}\cos\Delta\phi_{ab2} \\ &- \cos\Delta\phi_{ij2}\cos\Delta\phi_{ab1}\sin\Delta\phi_{ab2}). \quad (1c) \end{aligned}$$

Here $\Delta\phi_{ab1}$, $\Delta\phi_{ab2}$, and $\Delta\phi_{ab3}$ are the accumulated phase differences between normal modes in the interferometers and $\Delta\phi_{ij1}$, $\Delta\phi_{ij2}$, and $\Delta\phi_{ij3}$ are the accumulated phase differences between normal modes in the directional couplers. Applying drive voltages to the interferometers as discussed above we take

$$\begin{aligned} \Delta\phi_{ab1} &= \phi \\ \Delta\phi_{ab2} &= -\phi \\ \Delta\phi_{ab3} &= \phi \end{aligned} \tag{2}$$

where ϕ represents a phase difference proportional to applied voltage.

Equation 1 can then be expressed as

$$P = \tfrac{1}{2}(1 + f_1\sin\phi - f_2\sin 2\phi + f_3\sin 3\phi) \tag{3}$$

where

$$\begin{aligned} f_1 =& \cos\Delta\phi_3\sin\Delta\phi_1\cos\Delta\phi_2 + \frac{\sin\Delta\phi_3}{4} \\ &\cdot(1 + 3\cos\Delta\phi_1 - \cos\Delta\phi_2 + \cos\Delta\phi_1\cos\Delta\phi_2) \\ f_2 =& \frac{\sin\Delta\phi_2}{2} \\ &\cdot(\cos\Delta\phi_3 - \cos\Delta\phi_3\cos\Delta\phi_1 + \sin\Delta\phi_3\sin\Delta\phi_1) \\ f_3 =& \frac{\sin\Delta\phi_3}{4} \\ &\cdot(1 - \cos\Delta\phi_1 - \cos\Delta\phi_2 + \cos\Delta\phi_1\cos\Delta\phi_2). \end{aligned} \tag{4}$$

In (4) we have replaced $\Delta\phi_{ij1}$ with $\Delta\phi_1$, etc. By using the Taylor series expansion for $\sin\phi$

$$\sin\phi = \phi - \frac{\phi^3}{6} + \frac{\phi^5}{120} - \frac{\phi^7}{5040} + \cdots \tag{5}$$

in (3) we cast (3) in the form

$$P = \tfrac{1}{2}(1 + d_1\phi + d_3\phi^3 + d_5\phi^5 + d_7\phi^7 + \cdots) \tag{6}$$

with the coefficients

$$\begin{aligned} d_1 &= f_1 - 2f_2 + 3f_3 \\ d_3 &= \tfrac{1}{6}(-f_1 + 8f_2 - 27f_3) \\ d_5 &= \tfrac{1}{120}(f_1 - 32f_2 + 243f_3) \\ d_7 &= \tfrac{1}{5040}(-f_1 + 128f_2 - 2187f_3). \end{aligned} \tag{7}$$

The problem becomes one of maximizing d_1 (for maximum modulator efficiency) while setting $d_3 = d_5 = 0$. This yields the phase differences $\Delta\phi_1$, $\Delta\phi_2$, and $\Delta\phi_3$ in the three directional couplers, which in turn will define their coupling coefficients. Equation (7) with $d_3 = d_5 = 0$ yields

$$d_1 = 2f_1/3$$

and

$$\begin{aligned} f_1 - 5f_2 &= 0 \\ f_1 - 45f_3 &= 0. \end{aligned} \tag{8}$$

There appears to be no analytical solution to this problem so we use a numerical approach. Substituting $x_1 = \sin\Delta\phi_1$ and $y_1 = \cos\Delta\phi_1$, etc. in (8b) and solving for x_3/y_3 yields

$$\frac{x_3}{y_3} = \frac{-2[2x_1y_2 - 5x_2(1 - y_1)]}{1 + 3y_1 - y_2 + y_1y_2 - 10x_1x_2}. \tag{9}$$

Similarly (8c) yields

$$\frac{x_3}{y_3} = \frac{-x_1y_2}{-11 + 12y_1 + 11y_2 - 11y_1y_2}. \tag{10}$$

Equating (9) and (10) and solving for x_2, y_2 in terms of x_1, y_1, yields

$$x_2(a_2 + a_4y_2) + a_3y_2(1 - y_2) = 0 \tag{11}$$

where

$$\begin{aligned} a_2 &= 10(11 - 23y_1 + 12y_1^2) \\ a_3 &= 45x_1(y_1 - 1) \\ a_4 &= 10(-11 + 22y_1 - 11y_1^2 + x_1^2). \end{aligned} \tag{12}$$

Equations (11) and (12) can be solved numerically for $\Delta\phi_2$ as a function of $\Delta\phi_1$, and the solutions inserted into (9) or (10) to get $\Delta\phi_3$. These values can then be inserted into (4a) and (8a) to get the slope of the quasilinear function given by (6). In doing this it was observed that a maximum in slope, $d_1 = 2f_1/3$, occurred for $\Delta\phi_3 = \pi/2$. By assuming this value and substituting in (4), (8b), and (8c) the variables x_1, x_2, and y_2 can be eliminated and a cubic equation in y_1 obtained

$$22 - 43.25y_1 + 18.5y_1^2 + 2.75y_1^3 = 0. \tag{13}$$

The appropriate root is $y_1 = 0.92465$ which yields

$$\begin{aligned} \Delta\phi_1 &= 0.39068 \\ \Delta\phi_2 &= 1.68664 \\ \Delta\phi_3 &= \frac{\pi}{2} \end{aligned} \tag{14}$$

in radians, and the maximum value of $f_1 = 0.945665$. Then from (7) we obtain

$$\begin{aligned} d_1 &= 0.630444 \\ d_3 &= -6 * 10^{-7} \\ d_5 &= 7 * 10^{-8} \\ d_7 &= -0.00450316 \end{aligned} \tag{15}$$

showing the coefficients d_3 and d_5 are ~ 0.

Using these coefficients (6) is plotted in Fig. 2(a). In Fig. 2(b) this result is compared to a straight line and a sine function

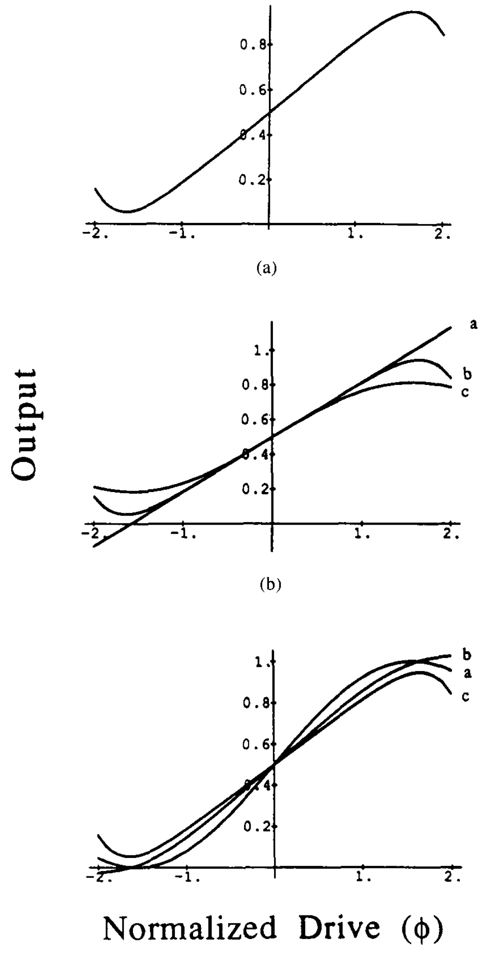

Fig. 2. (a) Transfer function of the three section linearized modulator. (b) Linearized transfer function compared to linear and sine functions with the same slope. a) 0.5(1+0.6304ϕ), b) linearized transfer function, c) 0.5(1+0.6304 sin ϕ). (c) Transfer functions for [a] single section, [b] two section (solution from [6]), and [c] three-section cascaded modulator.

with the same slope. In Fig. 2(c) it is compared to the optimum transfer characteristics of a single section (sine function) and a two section (solution from [6]) device. The improvement in linearity of the three section device occurs at a small loss in slope efficiency compared to the single and two section devices.

III. Harmonic Distortion

Another way to characterize performance of this device is to calculate the harmonic distortion level. If we apply a three tone signal defined by

$$\phi = m(\sin\omega_1 t + \sin\omega_2 t + \sin\omega_3 t) \tag{16}$$

and substitute in (6), we obtain

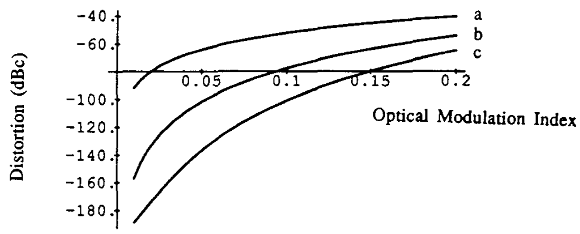

Fig. 3. Harmonic distortion for the $\sin(\omega_1 \pm \omega_2 \pm \omega_3)t$ term versus OMI($= md_1$) for the a) single section, b) two section (solution from [6]), and c) three section modulators.

$$\begin{aligned} P = \tfrac{1}{2}\{1 &+ (md_1 + \tfrac{15}{4}m^3d_3 + \tfrac{155}{8}m^5d_5 + \tfrac{7455}{64}m^7d_7 + \cdots) \\ &\cdot(\sin\omega_1 t + \sin\omega_2 t + \sin\omega_3 t) \\ &- (\tfrac{3}{2}m^3d_3 + \tfrac{45}{4}m^5d_5 + \tfrac{315}{4}m^7d_7 + \cdots) \\ &\cdot[\sin(\omega_1 \pm \omega_2 \pm \omega_3)t + \cdots]t + \cdots\} \end{aligned} \tag{17}$$

where in (17) we show explicitly only the coefficient of the fundamental terms and the largest harmonic term, which is the third harmonic due to the $\sin(\omega_1 \pm \omega_2 \pm \omega_3)t$ term. This term has contributions from the fifth and higher order Taylor series terms, as well as the third order Taylor series term. We define harmonic distortion in dB's below the carrier (dBc) as

$$10\ \log\left[\frac{P_{ele}(harmonic)}{P_{ele}(carrier)}\right] \tag{18}$$

where electrical harmonic signal and carrier powers are employed. Typically harmonic distortion is plotted versus optical modulation index (OMI), which we define here as md_1. We show this plot in Fig. 3 for the largest third order harmonic term $\sin(\omega_1 \pm \omega_2 \pm \omega_3)t$, for the single, two, and three section devices. For the three section device (18) becomes

$$20\ \log\left(\frac{\tfrac{3}{2}m^3d_3 + \tfrac{45}{4}m^5d_5 + \tfrac{315}{4}m^7d_7 + \cdots}{md_1}\right) \tag{19}$$

where we have assumed the electrical power to be proportional to the square of the optical power. Fig. 3 shows that the three section device typically has ~20 dB better performance then the two section device, which in turn is ~20 dB better then the single section device.

IV. Discussion and Conclusion

Going back to our original assumption on drive voltages (1:−1:1), it is possible to show by direct calculation that the other assumption of voltages applied in the ratio 1:−1:−1 does not provide a solution.

The coupling coefficients for the directional couplers are obtained from the relation $\Delta\phi/2 = L\kappa$, where κ is the coupling coefficient and L is the directional coupler length. We are assuming that the directional couplers are synchronous, i.e., that the channels have the same propagation constants. If this is not the case, for example if the couplers are tuned electrically, we have

$$\frac{\Delta\phi}{2} = L\sqrt{\delta^2 + \kappa^2} \tag{20}$$

where $\delta = (\beta_1 - \beta_2)/2$ and β_1 and β_2 are the propagation constants of the waveguides in the directional coupler. For $\delta \neq 0$ additional phase terms arise which have been omitted in (1). These terms can be compensated for by the appropriate application of bias voltages to the interferometers as shown in [6], so that the problem reduces to the one treated here.

The solution obtained here with $\Delta\phi_3 = \pi/2$ implies that the final directional coupler is a 3-dB coupler. The solution appears to be the proper optimized solution for the structure, judging from the behavior of the transfer functions for the one, two and three section structures. However the derivation provided here does not absolutely determine that there is not some other more optimum solution for the structure. We have made the implicit assumption that the derivation in [6] which recommends alternating sign, equal magnitude drives for the two section device will equally apply to higher order devices. The fact that we have obtained an apparently optimized solution supports this assumption. Whether the symmetry associated with the final coupler has some physical rationale, or is just a fortunate coincidence which led to the current solution is not clear. The solution for the two section structure offers no guidance as it shows no such symmetry.

We have given a solution to the three section serially cascaded Mach Zehnder device which is the first to eliminate both the third and fifth order Taylor series terms. It results in about 20-dB improvement in third harmonic distortion compared to the two section device. In terms of OMI, the OMI is increased by ~50% for a given level of harmonic distortion, as shown in Fig. 3. For cable TV operation this improvement in linearity would result in a substantial increase in system capacity supplied by a single modulator, thus decreasing system costs.

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